ROLLING VERSUS SLIDING IN DYNAMICAL SYSTEMS PART I: THEORETICAL ASPECTS AND NUMERICAL SIMULATIONS

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Abstract: *The paper presents a synthesis of the most important friction models found in literature. The work continues with a presentation of the utmost significant aspects and the utility of dynamic analysis programs for the study of mechanical systems. A simulation of a mechanical system consisting of a cylinder that descends on an inclined plane was made. Several conditions were imposed for the friction between the contacting surfaces and the occurrence of pure rolling motion was highlighted.*
Keywords: *friction, pure rolling, contact point, COF*

1. Introduction

In mechanical engineering, the vast majority of interactions between two solid bodies can be modelled as a point contact between Σ_1 and Σ_2, boundary surfaces of the two bodies, Figure 1, [1]. It is accepted that in the theoretical point of contact C, both surfaces are regular so that at this point the common normal $\boldsymbol{n}$ and the common tangent plane P are well defined. The relative motion between the two bodies is defined by the relative velocity vectors $\boldsymbol{v}_r$ and the angular velocity vector ω. While the velocity vector is contained in the common tangent plane, the angular velocity vector has a certain direction and can be decomposed into two components: ω_r - the rolling angular velocity contained in the tangent plane and ω_s -the spin angular velocity parallel to the normal $\boldsymbol{n}$. The torsor of the reactions from the contact point contains: the normal reaction $\boldsymbol{N}$, the friction force $\boldsymbol{T}$ opposing to the relative velocity and the rolling and spin moments, $\boldsymbol{M}_r$ and $\boldsymbol{M}_s$, opposing to the rolling angular velocity and the spin angular velocity ω_s, respectively. The sliding friction is a factor for noises and energy consumption, which is why, it is desirable to replace it with rolling friction when is possible. Obviously, this is not always possible. An example for this situation concerns the cam mechanisms.

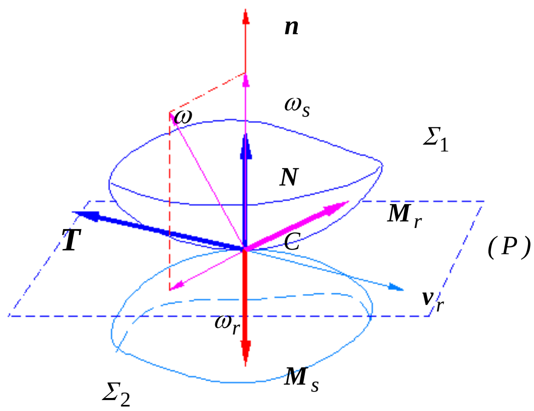
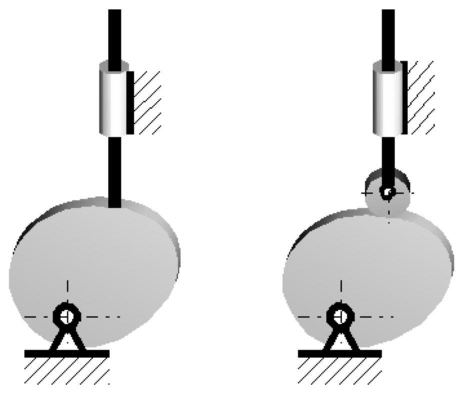


Figure 1: *The interaction between two solid bodies.*



a) b)

Figure 2: *Examples of cam mechanisms: a) cam and follower with tip; b) cam and follower with roller.*

Thus, if for a mechanism with cam and follower with tip Figure.2a, due to the fixed position of the contact point on the follower, a roller can be placed on the tip of the follower, Figure 2b, which transforms the sliding friction into rolling friction. The cam mechanisms with flat-faced follower, Figure 3, this is no longer possible due to the fact that the contact point is mobile, both on the cam and on the follower.

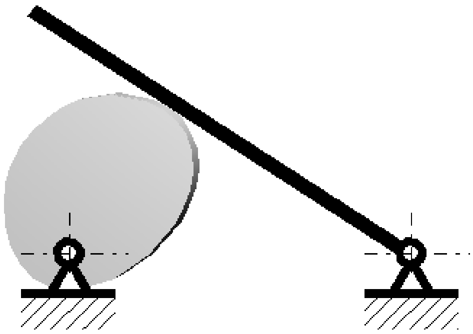


Figure 3: *Cam mechanism with flat-faced follower.*

2. Conditions for pure rolling existence

From the kinematic point of view, pure rolling occurs when the relative velocity $\boldsymbol{v}_r$ is zero. If there is dry friction between the contacting surfaces, the rolling condition can be expressed by the relation that must be accomplished between the frictional force $\boldsymbol{F}$ and the normal $\boldsymbol{N}$.

$$\left|\frac{T}{N}\right| < \mu_{st} \qquad (1)$$

where, μ_{st} is the static friction coefficient.

Figure. 4 reveals the simplest Coulomb friction model characterized by the variation of the friction coefficient as a function of the relative speed, [2, 3].

The necessity of modelling, as faithfully as possible, the behaviour of dynamic systems in which dry friction occurs conducts to much more complex friction models characterized by many more parameters such as: Bengisu-Akai, Dahl, LuGre, [4].

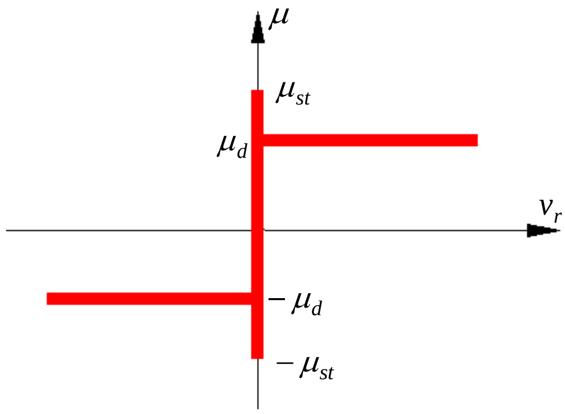


Figure 4: *The correlation between COF and relative velocity.*

Given the vector definition module of the dynamic friction force,

$$\boldsymbol{T} = -\mu_d N\, sgn(v_r)\boldsymbol{u}_{vr} \qquad (2)$$

where, $\boldsymbol{u}_{vr}$ is the versor of the relative velocity from the contact point,

$$\mathbf{u}_{vr} = \frac{\mathbf{v}_r}{|\mathbf{v}_r|}, \qquad (3)$$

and $sgn(x)$ is the signum function defined by:

$$\operatorname{sgn}(x) = \begin{cases} 1, & if\ x > 0 \\ -1, & if\ x < 0 \\ 0, & if\ x = 0 \end{cases}. \qquad (4)$$

It is expected that the differential equations of a dynamic system in which there is dry friction to be strongly nonlinear and due to this fact, must be integrated by numerical methods.

3. Dynamic system proposal and its dynamic simulation

Another consequence of the need for quick solutions to dynamics problems led to the appearance of dedicated software. One of these software is the MSC ADAMS, one of the most advanced and complex dynamic analysis software.

A particularly thorny problem in the use of dynamic software consists in the need to provide an important number of parameters to characterize the adopted tribological model, [5].

Subsequent, we consider one of the simplest problems of rigid dynamics in which the occurrence of rolling with sliding phenomena is possible. An homogeneous cylinder of mass M, radius r, height h, descending on an inclined plane (α the angle of the inclined plane) is simulated.

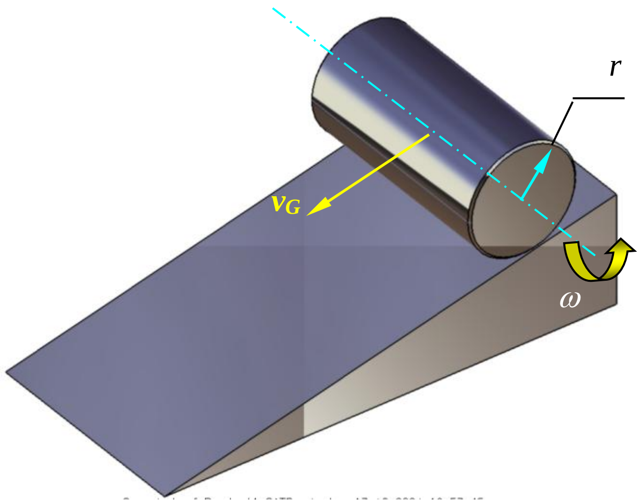


Figure 5: *The cylinder descending on the inclined plane*

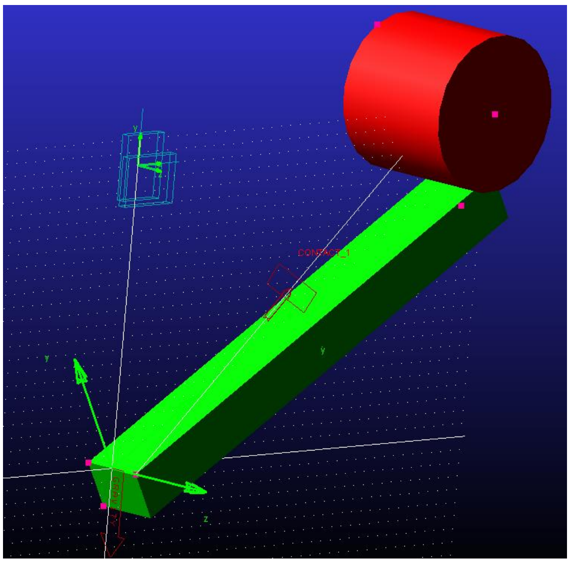


Figure 6: *The simulation of the mechanical system obtained by using MSC. ADAMS software .*

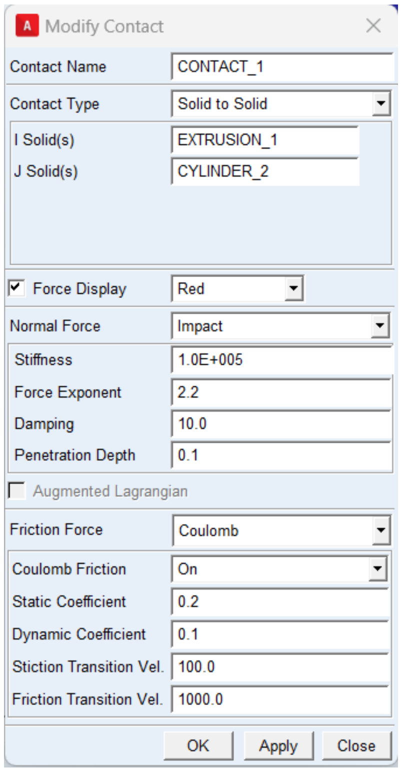


Figure 7: *The imposed values for the tribological parameters.*

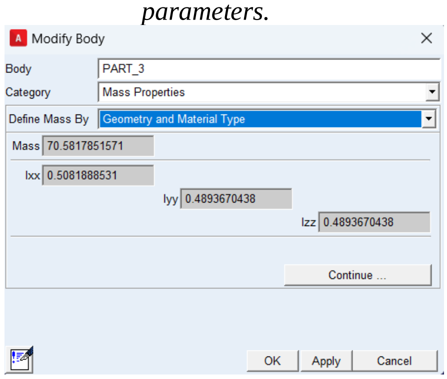


Figura 8: *Inertial parameters of the modelled cylinder.*

4. Obtained results using MSC. ADAMS software

For the above presented values of the tribological parameters defined in Figure 7 and with the values of the inertial characteristics defined in Figure 8, the variation of the angular speed and the displacement speed of the axis of the cylinder v_G, were determined corresponding to 30^o inclination angle of the plane. In Figure 9 and 10 are presented these dependencies for three different tribological situations, defined by using the values noted in the windows on the graphs. The first value represents the static friction coefficient μ_{st} and the second value represents the dynamic friction coefficient μ_d .

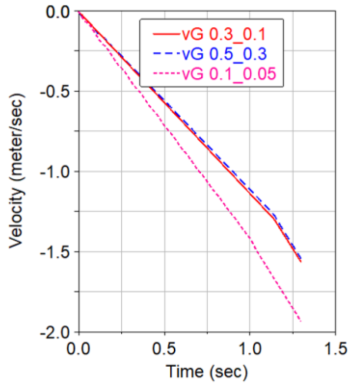


Figure 9: *The variation of the cylinder velocity.*

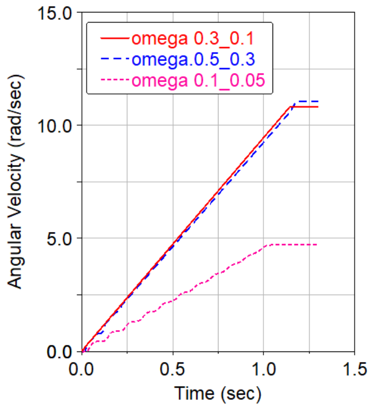


Figure 10: *The variation of the cylinder angular velocity.*

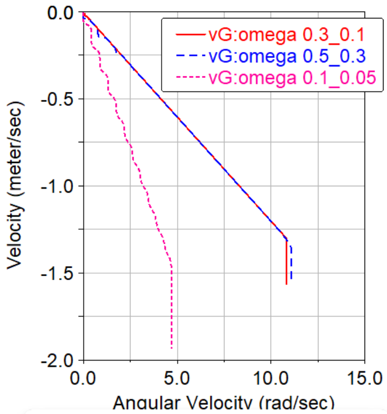


Figure 11: *Testing the pure rolling condition using the dependence between the velocity of the center of mass and the angular velocity.*

In order to highlight the cases in which the displacement of the cylinder was obtained by pure rolling, in Figure 11, a graphical correlation between the velocity of the displacement of the cylinder axis and the angular velocity was represented. It can be observed that the graphs for the first two tribological situations are identical and they are represented by a straight line that passes through the origin (this behaviour attests to the presence of pure rolling), while for the last case, corresponding to the reduced values of the friction coefficient, the fork deviates from the rectilinear shape.

5. Conclusions

In the first part of the paper, a review of the components of the reaction torsor in the case of single point concentrated contacts is carried out, and the installation conditions of the pure rolling are presented.

Next, the MSC.ADAMS software intended for the dynamic simulation of mechanical systems is briefly presented. With the help of this software, a simulation of the descent of a cylinder on the line of greatest slope of an inclined plane is obtained. The simulation was carried out for three pairs of values of the static and dynamic friction coefficients. The obtained results reveal condition of the pure rolling occurrence.

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