EXPERIMENTAL DEVICE FOR THE STUDY OF DRY FRICTION

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Abstract: *This paper presents an experimental device intended for the study of dry friction between a cylindrical surface and a sphere. The cylindrical surface is materialised by a physical pendulum whose movement is monitored using a wireless rotation sensor, which measures and plots the graph of the angle variation as a function of time. The sphere is materialised by a horizontal lever that presses on the pendulum by its own weight.*

The main purpose of the paper is to find the coefficient of friction between the physical pendulum and the lever by imposing the condition that the theoretical model and the experimental pendulum have identical decreases in angular amplitude.

Keywords: *friction coefficient, physical pendulum, nonlinear differential solution*

1. Introduction

Unlike the simple pendulum, which is an idealised model with a point mass and an inextensible string, the physical pendulum is a real system, consisting of a solid body that oscillates around a horizontal axis that does not pass through its centre of mass (centre of gravity) and on which only its own weight acts [Baker, 2005] , [Meriam, 2011].

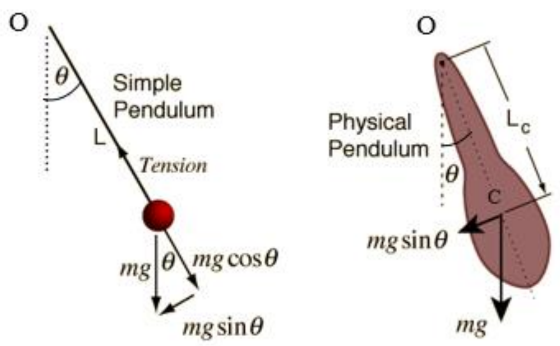

Figure 1: *Simple pendulum vs. physical pendulum*

The rigid body has uniformly distributed mass, such that the centre of gravity C is at a distance L from the point of rotation, Fig.2. The total weight of the rod, denoted by *mg*, acts at point C. In order to analyse the moments acting on the system, the force mg is broken down into two components: one component parallel to the rod, $mgcos\theta$, which does not contribute to the rotational movement and another component, $mgsin\theta$, which generates a moment relative to point O, favouring the oscillation of the pendulum.

The moment of the gravitational force acting on the pendulum can be expressed as follows:

$$M_O(G) = -mgLsin\theta. \quad (1)$$

Where:

- θ is the angular displacement of the pendulum from the vertical position
- g is the gravitational acceleration;
- *L* is the distance from the axis of rotation to the centre of mass of the pendulum;
- m is the mass of the pendulum.

Applying the moment of momentum theorem

$$\frac{d\bar{K}_O}{dt} = \bar{M}_O(\bar{G}). \quad (2)$$

We obtain the differential equation of the motion of the physical pendulum around a fixed axis:

$$J_O \frac{d^2\theta}{dt^2} + mgL\sin\theta = 0. \qquad (3)$$

Where:

- J_O is the mass moment of inertia of the pendulum relative to the point of oscillation O
- $d^2\theta / dt^2$ is the angular acceleration.

For small oscillations, the period of a physical pendulum is obtained, [Sartijito, 2020]:

$$T = 2\pi\sqrt{\frac{J_O}{mgL}}. \qquad (4)$$

2. Experimental setup

To study dry friction between a spherical and a cylindrical surface (steel-steel), the device shown in Fig. 2 was constructed.

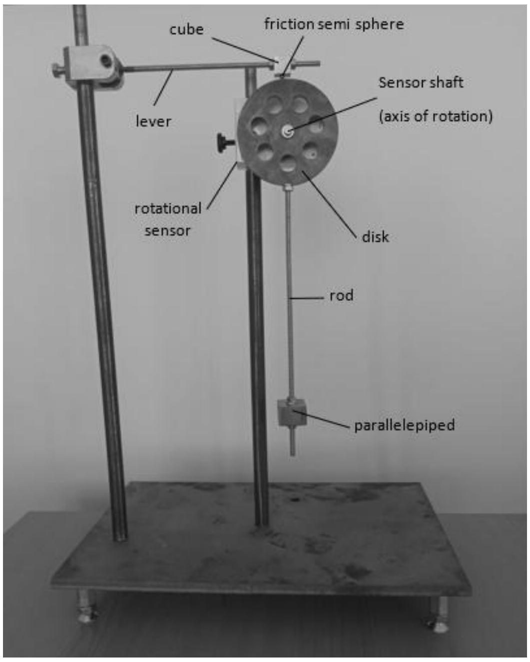

Figure 2: *Experimental setup*

The cylindrical surface is represented by a physical pendulum fixed on the axis of a wireless rotation sensor, which allows experimental data to be acquired. The pendulum consists of a disc with seven holes, a vertical rod and a parallelepiped body. The parallelepiped body can be moved at different distances from the axis of rotation, thus obtaining different values for the mass moment of inertia.

The spherical surface is embedded in a cubic body fixed on a horizontal lever that presses on the pendulum, creating dry friction contact.

The oscillatory motion of the pendulum is monitored by the Pasco sensor, Fig. 3, which records the kinematic parameters and plots the variation of the swinging angle as a function of time, [Butcher, 2016].

Figure 3: *Wireless rotation sensor*

3. Differential equation of motion

To analyse dry friction between a hemisphere and a cylinder, a calculation program is developed in Mathcad to plot the graph of the angular displacement of the pendulum, $\theta(t)$.

The input parameters were determined: the mass of the pendulum, the normal pressing force, the position of the centre of mass of the pendulum and the mass moment of inertia.

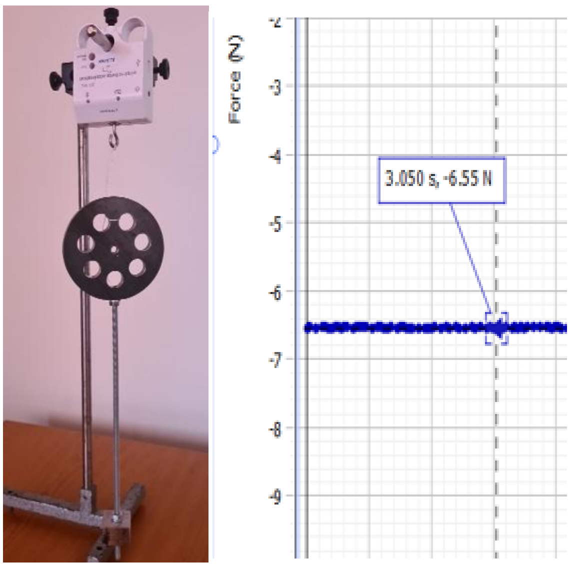

Figure 4: *Pendulum weight*

A wireless force sensor was used to measure the weight of the pendulum consisting of a disc, a rod and attached parallelepiped.

The weight of the pendulum is 6.55 N, Fig.4. The pressing force was determined in a similar way, being equal to the weight of the horizontal lever (rod plus attached cube), Fig.5.

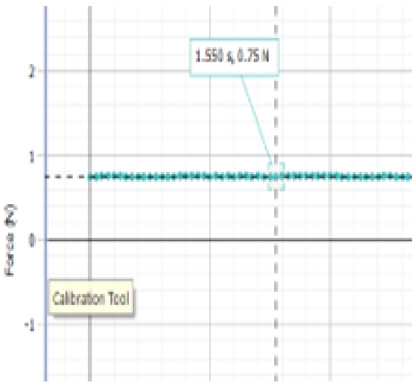

Figure 5: *Normal pressing force*

The pendulum used is an assembly consisting of a disc with seven holes, a threaded rod and a parallelepiped body attached to it.

The position of its centre of mass and the moment of inertia relative to the axis of rotation were determined using Autodesk Inventor software, considering Oy as the axis of symmetry of the pendulum, Fig.6.

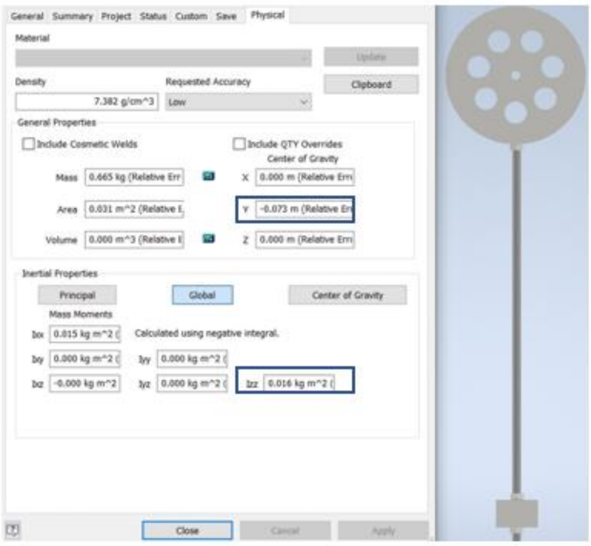

Figure 6: *Mass characteristics of the physical pendulum (Autodesk Inventor)*

The total mass moment of inertia is given by the relation:

$$J_O = J_{disc} + J_{tij\breve{a}} + J_{paralelipiped}. \qquad (5)$$

The following values were obtained: y_C=0.073 m and J_O = 0.016 kg·m^2.

The pendulum is acted upon vertically by its own weight mg and the normal pressing force Fap exerted by the horizontal lever through the spherical surface. In addition, due to the dry friction between the sphere and the circumference of the disc, a friction force μF_{ap} appears, which opposes the movement of the disc and causes the damping of the oscillations.

Applying the kinetic momentum theorem, we obtain the nonlinear differential equation of the pendulum's motion:

$$J_O \ddot{\theta} = -mg\sin\theta - \mu F_{ap} r\, sign(\dot{\theta}). \qquad (6)$$

Where:

- $\ddot{\theta}$ is the angular acceleration;
- μ is the steel-steel friction coefficient between the horizontal lever and the pendulum disc;
- F_{ap} is the normal pressing force given by the weight of the horizontal lever
- r is the radius of the disc;
- $sign(\dot{\theta})$ takes into account the direction of angular velocity.

Given the nonlinearity of Eq. (4), it cannot be solved analytically. The law of variation of the oscillation angle was obtained in Mathcad, using the 4th order Runge-Kutta method, [Alaci, 2019], considering that the pendulum is launched from rest with an initial angular amplitude. The graph of the variation of the swinging angle θ as a function of time was obtained, Fig.7.

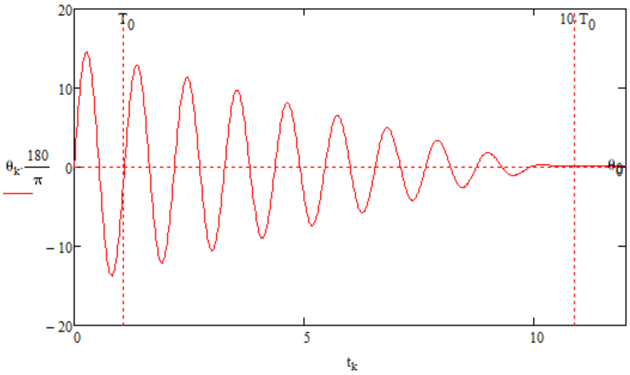

Figure 7: *Graph of swinging angle (degree) versus time (s)(result of calculation- Mathcad)*

4. Experimental results

Using the rotation sensor and Pasco Capstone software, data acquisition was initially performed for free oscillations, i.e. without pressing force, Fig.8.

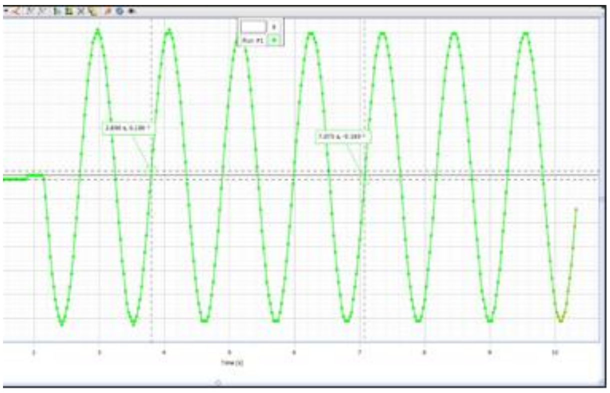

Figure 8: *Free oscillations. Graph of swinging angle (degree) versus time (s)(result of experiment).*

The graph of swinging angle (degree) versus time (s), θ(t) shows the absence of damping, therefore the friction moment in the sensor axis has a negligible value.

The second experiment was performed with the horizontal lever pressing on the outer surface of the penduluma with a force F_{ap}=0.75N.

The experimental data in this case are shown in the graph of damped oscillations from Fig. 9.

A linear decrease in the oscillation amplitude over time is observed.

The experimental data from Capstone were exported to Mathcad in order to compare them with the theoretical model, Fig. 10.

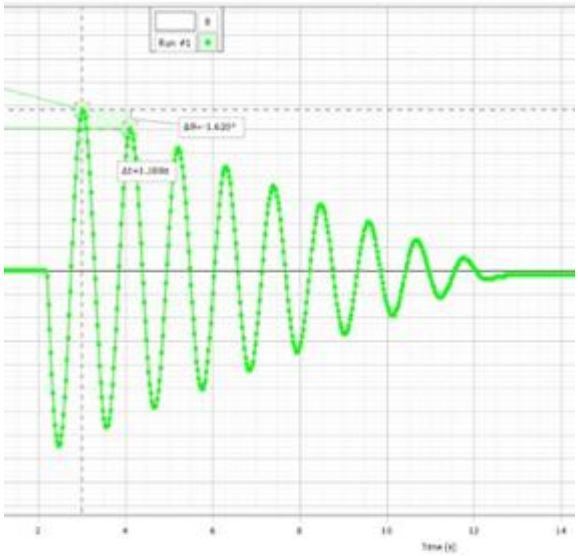

Figure 9: *Damped oscillations. Graph of swinging angle (degree) versus time (s)(result of experiment).*

By superimposing the two graphs and imposing the condition of obtaining the same linear damping rate of amplitude, [6], the value of the steel-steel friction coefficient μ =0.097 was obtained.

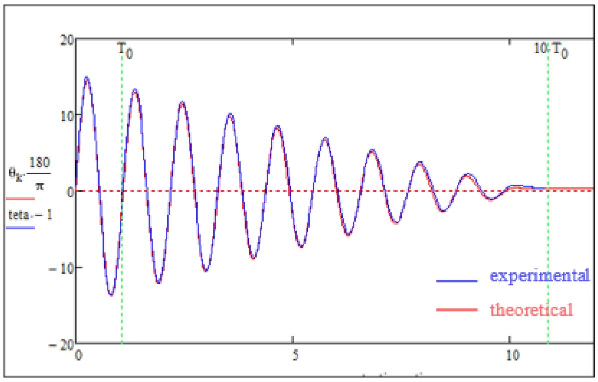

Figure 10: *Swinging angle versus time (overlap theoretical and experimental)*

4. Conclusions

The paper presents a method and the corresponding device used to determine the coefficient of sliding friction. The instrument used is a physical pendulum with a cylindrical external surface as the contact surface. It comes into contact with a fixed spherical surface, thus achieving point contact.

Starting from the kinetic momentum theorem, the differential equation of the pendulum's oscillatory motion was obtained. This was numerically integrated using the Runge Kutta method, and a linear damping of the pendulum's amplitude was obtained. The theoretical values of the oscillation angle are compared with the experimental results obtained using a wireless rotation sensor. The friction coefficient is determined by comparing the results of the theoretical model and the experimental data and imposing the condition that both have identical decreases in angular amplitude.

5. References

1. [Alaci, 2019] S Alaci, R-D Pentiuc, F C Ciornei, I C Romanu, L Irimescu and F Buium, Employment of hyper-cycloidal oscillatory motion for finding the coefficient of rolling friction. Part 1: Theoretical model, 2019 IOP Conf. Ser.: Mater. Sci. Eng. 514 012002.
2. [Baker, 2005] G. L. Baker and J. A. Blackburn, *The Pendulum: a case study in physics*, ed. 1. New York NY: Oxford University Press, 2005.
3. [Butcher, 2016] J. C. Butcher, Numerical Methods for Ordinary Differential Equations, 3rd ed. Chichester, England: John Wiley & Sons, 2016.
4. [Meriam, 2011] Meriam, J. L., & Kraige, L. G. (2011). Engineering Mechanics: Dynamics (7th ed.). Wiley
5. [Sartijito, 2020] S. Sartijito and N. Yuningsih, “The Period of Physical Pendulum Motion with Large Angular Displacement”, Advances in Engineering Research, vol. 198, International Seminar on Science and Applied Technology (ISSAT 2020), pp. 198-199, 2020.
6. PASCO Scientific, “Wireless Rotary Motion Sensor,” PASCO Scientific, 2024.